3 Differential Coding

Instead of encoding a signal directly, the *differential coding* technique codes the difference between the signal itself and its prediction. Therefore it is also known as *predictive coding*. By utilizing spatial and/or temporal interpixel correlation, differential coding is an efficient and yet computationally simple coding technique. In this chapter, we first describe the differential technique in general. Two components of differential coding, prediction and quantization, are discussed. There is an emphasis on (optimum) prediction, since quantization was discussed in Chapter 2. When the difference signal (also known as prediction error) is quantized, the differential coding is called differential pulse code modulation (DPCM). Some issues in DPCM are discussed, after which delta modulation (DM) as a special case of DPCM is covered. The idea of differential coding involving image sequences is briefly discussed in this chapter. More detailed coverage is presented in Sections III and IV, starting from Chapter 10. If quantization is not included, the differential coding is referred to as information-preserving differential coding. This is discussed at the end of the chapter.

3.1 INTRODUCTION TO DPCM

As depicted in Figure 2.3, a source encoder consists of the following three components: transformation, quantization, and codeword assignment. The transformation converts input into a format for quantization followed by codeword assignment. In other words, the component of transformation decides which format of input is to be encoded. As mentioned in the previous chapter, input itself is not necessarily the most suitable format for encoding.

Consider the case of monochrome image encoding. The input is usually a 2-D array of gray level values of an image obtained via PCM coding. The concept of spatial redundancy, discussed in Section 1.2.1.1, tells us that neighboring pixels of an image are usually highly correlated. Therefore, it is more efficient to encode the gray difference between two neighboring pixels instead of encoding the gray level values of each pixel. At the receiver, the decoded difference is added back to reconstruct the gray level value of the pixel. Since neighboring pixels are highly correlated, their gray level values bear a great similarity. Hence, we expect that the variance of the difference signal will be smaller than that of the original signal. Assume uniform quantization and natural binary coding for the sake of simplicity. Then we see that for the same bit rate (bits per sample) the quantization error will be smaller, i.e., a higher quality of reconstructed signal can be achieved. Or, for the same quality of reconstructed signal, we need a lower bit rate.

3.1.1 SIMPLE PIXEL-TO-PIXEL DPCM

Denote the gray level values of pixels along a row of an image as z_i, $i = 1,\cdots,M$, where M is the total number of pixels within the row. Using the immediately preceding pixel's gray level value, z_{i-1}, as a prediction of that of the present pixel, $\hat{z}_i$, i.e.,

$$\hat{z}_i = z_{i-1} \tag{3.1}$$

we then have the difference signal

$$d_i = z_i - \hat{z}_i = z_i - z_{i-1} \tag{3.2}$$

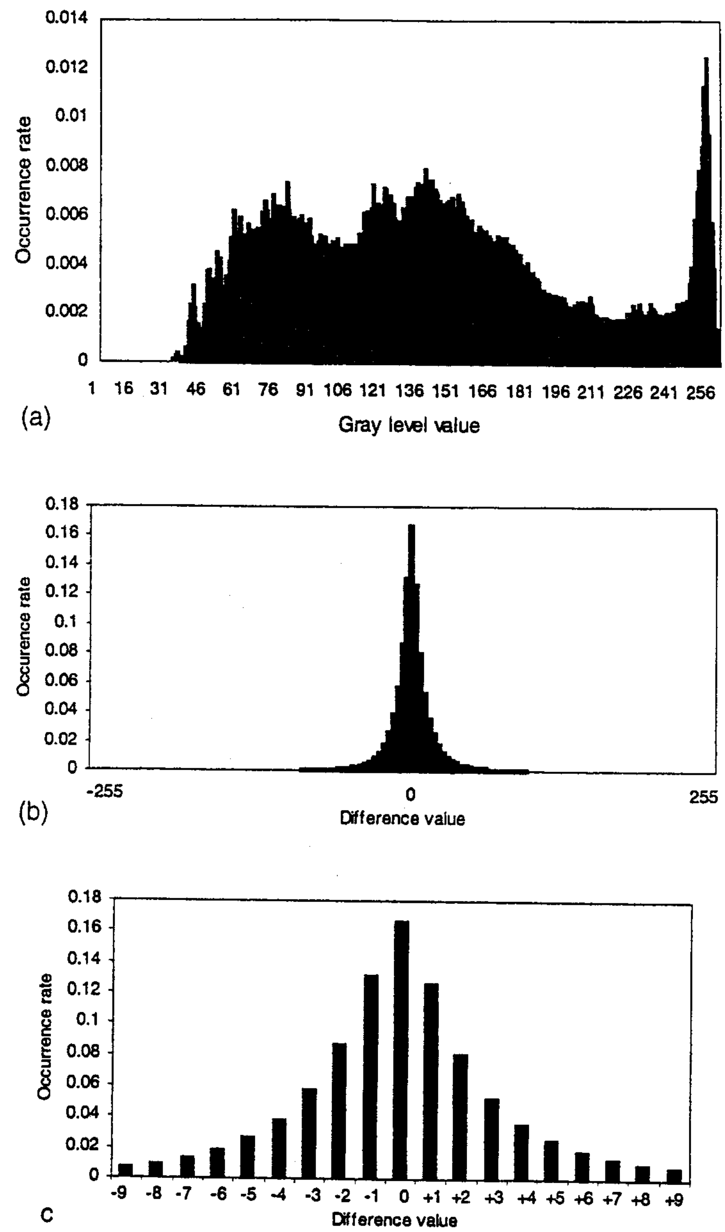
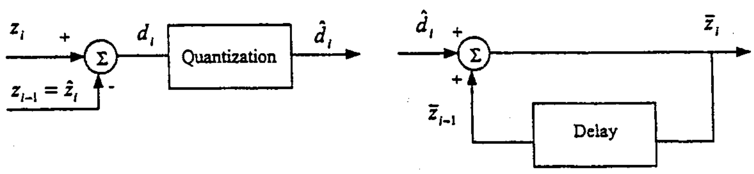


FIGURE 3.1 (a) Histogram of the original "boy and girl" image. (b) Histogram of the difference image obtained by using horizontal pixel-to-pixel differencing. (c) A close-up of the central portion of the histogram of the difference image.

Assume a bit rate of eight bits per sample in the quantization. We can see that although the dynamic range of the difference signal is theoretically doubled, from 256 to 512, the variance of the difference signal is actually much smaller. This can be confirmed from the histograms of the "boy and girl" image (refer to Figure 1.1) and its difference image obtained by horizontal pixel-to-pixel differencing, shown in Figure 3.1(a) and (b), respectively. Figure 3.1(b) and its close-up (c) indicate that by a rate of 42.44% the difference values fall into the range of −1, 0, and +1. In other words, the histogram of the difference signal is much more narrowly concentrated than that of the original signal.



(a) Encoder (b) Decoder

FIGURE 3.2 Block diagram of a pixel-to-pixel differential coding system.

A block diagram of the scheme described above is shown in Figure 3.2. There z_i denotes the sequence of pixels along a row, d_i is the corresponding difference signal, and $\hat{d}_i$ is the quantized version of the difference, i.e.,

$$\hat{d}_i = Q(d_i) = d_i + e_q \tag{3.3}$$

where e_q represents the quantization error. In the decoder, $\bar{z}_i$ represents the reconstructed pixel gray value, and we have

$$\bar{z}_i = \bar{z}_{i-1} + \hat{d}_i \tag{3.4}$$

This simple scheme, however, suffers from an accumulated quantization error. We can see this clearly from the following derivation (Sayood, 1996), where we assume the initial value z_0 is available for both the encoder and the decoder.

$$\text{as } i = 1, \quad d_1 = z_1 - z_0$$
$$\hat{d}_1 = d_1 + e_{q,1} \tag{3.5}$$
$$\bar{z}_1 = z_0 + \hat{d}_1 = z_0 + d_1 + e_{q,1} = z_1 + e_{q,1}$$

Similarly, we can have

$$as \quad i = 2, \quad \bar{z}_2 = z_2 + e_{q,1} + e_{q,2} \tag{3.6}$$

and, in general,

$$\bar{z}_i = z_i + \sum_{j=1}^{i} e_{q,j} \tag{3.7}$$

This problem can be remedied by the following scheme, shown in Figure 3.3. Now we see that in both the encoder and the decoder, the reconstructed signal is generated in the same way, i.e.,

$$\bar{z}_i = \bar{z}_{i-1} + \hat{d}_i \tag{3.8}$$

and in the encoder the difference signal changes to

$$d_i = z_i - \bar{z}_{i-1} \tag{3.9}$$

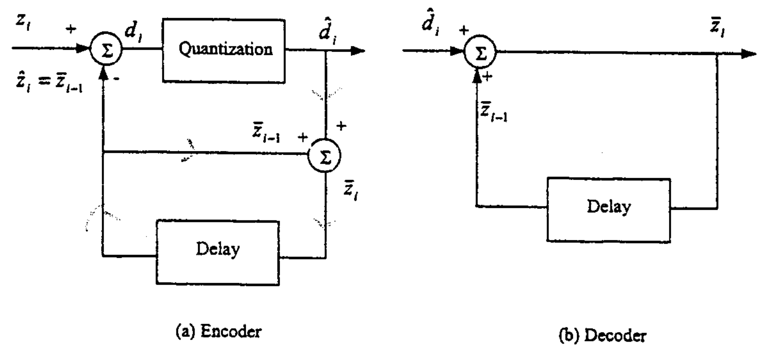


FIGURE 3.3 Block diagram of a practical pixel-to-pixel differential coding system.

Thus, the previously reconstructed $\bar{z}_{i-1}$ is used as the predictor, $\hat{z}_i$, i.e.,

$$\hat{z}_i = \bar{z}_{i-1}. \tag{3.10}$$

In this way, we have

$$\begin{aligned} as \quad i = 1, \quad d_1 &= z_1 - z_0 \\ \hat{d}_1 &= d_1 + e_{q,1} \\ \bar{z}_1 = z_0 + \hat{d}_1 &= z_0 + d_1 + e_{q,1} = z_1 + e_{q,1} \end{aligned} \tag{3.11}$$

Similarly, we have

$$\begin{aligned} as \quad i = 2, \quad d_2 &= z_2 - \bar{z}_1 \\ \hat{d}_2 &= d_2 + e_{q,2} \\ \bar{z}_2 = \bar{z}_1 + \hat{d}_2 &= z_2 + e_{q,2} \end{aligned} \tag{3.12}$$

In general,

$$\bar{z}_i = z_i + e_{q,i} \tag{3.13}$$

Thus, we see that the problem of the quantization error accumulation has been resolved by having both the encoder and the decoder work in the same fashion, as indicated in Figure 3.3, or in Equations 3.3, 3.9, and 3.10.

3.1.2 General DPCM Systems

In the above discussion, we can view the reconstructed neighboring pixel's gray value as a prediction of that of the pixel being coded. Now, we generalize this simple pixel-to-pixel DPCM. In a general DPCM system, a pixel's gray level value is first predicted from the preceding reconstructed pixels' gray level values. The difference between the pixel's gray level value and the predicted value is then quantized. Finally, the quantized difference is encoded and transmitted to the receiver. A block

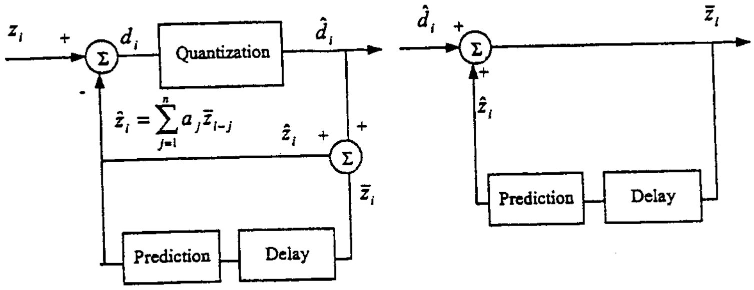


FIGURE 3.4 Block diagram of a general DPCM system.

diagram of this general differential coding scheme is shown in Figure 3.4, where the codeword assignment in the encoder and its counterpart in decoder are not included.

It is noted that, instead of using the previously reconstructed sample, $\bar{z}_{i-1}$, as a predictor, we now have the predicted version of z_i, $\hat{z}_i$, as a function of the n previously reconstructed samples, $\bar{z}_{i-1}, \bar{z}_{i-2}, \cdots, \bar{z}_{i-n}$. That is,

$$\hat{z}_i = f\left(\bar{z}_{i-1}, \bar{z}_{i-2}, \cdots, \bar{z}_{i-n}\right) \tag{3.14}$$

Linear prediction, i.e., that the function f in Equation 3.14 is linear, is of particular interest and is widely used in differential coding. In linear prediction, we have

$$\hat{z}_i = \sum_{j=1}^{n} a_j \bar{z}_{i-j} \tag{3.15}$$

where a_j are real parameters. Hence, we see that the simple pixel-to-pixel differential coding is a special case of general differential coding with linear prediction, i.e., $n = 1$ and $a_1 = 1$.

In Figure 3.4, d_i is the difference signal and is equal to the difference between the original signal, z_i, and the prediction $\hat{z}_i$. That is,

$$d_i = z_i - \hat{z}_i \tag{3.16}$$

The quantized version of d_i is denoted by $\hat{d}_i$. The reconstructed version of z_i is represented by $\bar{z}_i$, and

$$\bar{z}_i = \hat{z}_i + \hat{d}_i \tag{3.17}$$

Note that this is true for both the encoder and the decoder. Recall that the accumulation of the quantization error can be remedied by using this method.

The difference between the original input and the predicted input is called prediction error, which is denoted by e_p. That is,

$$e_p = z_i - \hat{z}_i \tag{3.18}$$

where the e_p is understood as the prediction error associated with the index i. Quantization error, e_q, is equal to the reconstruction error or coding error, e_r, defined as the difference between the original signal, z_i, and the reconstructed signal, $\bar{z}_i$, when the transmission is error free:

$$\begin{aligned} e_q &= d_i - \hat{d}_i \\ &= (z_i - \hat{z}_i) - (\bar{z}_i - \hat{z}_i) \\ &= z_i - \bar{z}_i = e_r \end{aligned} \tag{3.19}$$

This indicates that quantization error is the only source of information loss with an error-free transmission channel.

The DPCM system depicted in Figure 3.4 is also called closed-loop DPCM with feedback around the quantizer (Jayant, 1984). This term reflects the feature in DPCM structure.

Before we leave this section, let us take a look at the history of the development of differential image coding. According to an excellent early article on differential image coding (Musmann, 1979), the first theoretical and experimental approaches to image coding involving linear prediction began in 1952 at the Bell Telephone Laboratories (Oliver, 1952; Kretzmer, 1952; Harrison, 1952). The concepts of DPCM and DM were also developed in 1952 (Cutler, 1952; Dejager, 1952). Predictive coding capable of preserving information for a PCM signal was established at the Massachusetts Institute of Technology (Elias, 1955).

The differential coding technique has played an important role in image and video coding. In the international coding standard for still images, JPEG (covered in Chapter 7), we can see that differential coding is used in the lossless mode and in the DCT-based mode for coding DC coefficients. Motion-compensated (MC) coding has been a major development in video coding since the 1980s and has been adopted by all the international video coding standards such as H.261 and H.263 (covered in Chapter 19), MPEG 1 and MPEG 2 (covered in Chapter 16). MC coding is essentially a predictive coding technique applied to video sequences involving displacement motion vectors.

3.2 OPTIMUM LINEAR PREDICTION

Figure 3.4 demonstrates that a differential coding system consists of two major components: prediction and quantization. Quantization was discussed in the previous chapter. Hence, in this chapter we emphasize prediction. Below, we formulate an optimum linear prediction problem and then present a theoretical solution to the problem.

3.2.1 FORMULATION

Optimum linear prediction can be formulated as follows. Consider a discrete-time random process z. At a typical moment i, it is a random variable z_i. We have n previous observations $\bar{z}_{i-1}, \bar{z}_{i-2}, \cdots, \bar{z}_{i-n}$ available and would like to form a prediction of z_i, denoted by $\hat{z}_i$. The output of the predictor, $\hat{z}_i$, is a linear function of the n previous observations. That is,

$$\hat{z}_i = \sum_{j=1}^{n} a_j \bar{z}_{i-j} \tag{3.20}$$

with a_j, $j = 1,2,\cdots,n$ being a set of real coefficients. An illustration of a linear predictor is shown in Figure 3.5. As defined above, the prediction error, e_p, is

$$e_p = z_i - \hat{z}_i \tag{3.21}$$