2 Quantization

After the introduction to image and video compression presented in Chapter 1, we now address several fundamental aspects of image and video compression in the remaining chapters of Section I. Chapter 2, the first chapter in the series, concerns quantization. Quantization is a necessary component in lossy coding and has a direct impact on the bit rate and the distortion of reconstructed images or videos. We discuss concepts, principles and various quantization techniques which include uniform and nonuniform quantization, optimum quantization, and adaptive quantization.

2.1 QUANTIZATION AND THE SOURCE ENCODER

Recall Figure 1.1, in which the functionality of image and video compression in the applications of visual communications and storage is depicted. In the context of visual communications, the whole system may be illustrated as shown in Figure 2.1. In the transmitter, the input analog information source is converted to a digital format in the A/D converter block. The digital format is compressed through the image and video source encoder. In the channel encoder, some redundancy is added to help combat noise and, hence, transmission error. Modulation makes digital data suitable for transmission through the analog channel, such as air space in the application of a TV broadcast. At the receiver, the counterpart blocks reconstruct the input visual information. As far as storage of visual information is concerned, the blocks of channel, channel encoder, channel decoder, modulation, and demodulation may be omitted, as shown in Figure 2.2. If input and output are required to be in the digital format in some applications, then the A/D and D/A converters are omitted from the system. If they are required, however, other blocks such as encryption and decryption can be added to the system (Sklar, 1988). Hence, what is conceptualized in Figure 2.1 is a fundamental block diagram of a visual communication system.

In this book, we are mainly concerned with source encoding and source decoding. To this end, we take it a step further. That is, we show block diagrams of a source encoder and decoder in Figure 2.3. As shown in Figure 2.3(a), there are three components in source encoding: transformation, quantization, and codeword assignment. After the transformation, some form of an input information source is presented to a quantizer. In other words, the transformation block decides which types of quantities from the input image and video are to be encoded. It is not necessary that the original image and video waveform be quantized and coded: we will show that some formats obtained from the input image and video are more suitable for encoding. An example is the difference signal. From the discussion of interpixel correlation in Chapter 1, it is known that a pixel is normally highly correlated with its immediate horizontal or vertical neighboring pixel. Therefore, a better strategy is to encode the difference of gray level values between a pixel and its neighbor. Since these data are highly correlated, the difference usually has a smaller dynamic range. Consequently, the encoding is more efficient. This idea is discussed in Chapter 3 in detail.

Another example is what is called transform coding, which is addressed in Chapter 4. There, instead of encoding the original input image and video, we encode a transform of the input image and video. Since the redundancy in the transform domain is greatly reduced, the coding efficiency is much higher compared with directly encoding the original image and video.

Note that the term transformation in Figure 2.3(a) is sometimes referred to as *mapper* and *signal processing* in the literature (Gonzalez and Woods, 1992; Li and Zhang, 1995). Quantization refers to a process that converts input data into a set of finitely different values. Often, the input data to a quantizer are continuous in magnitude.

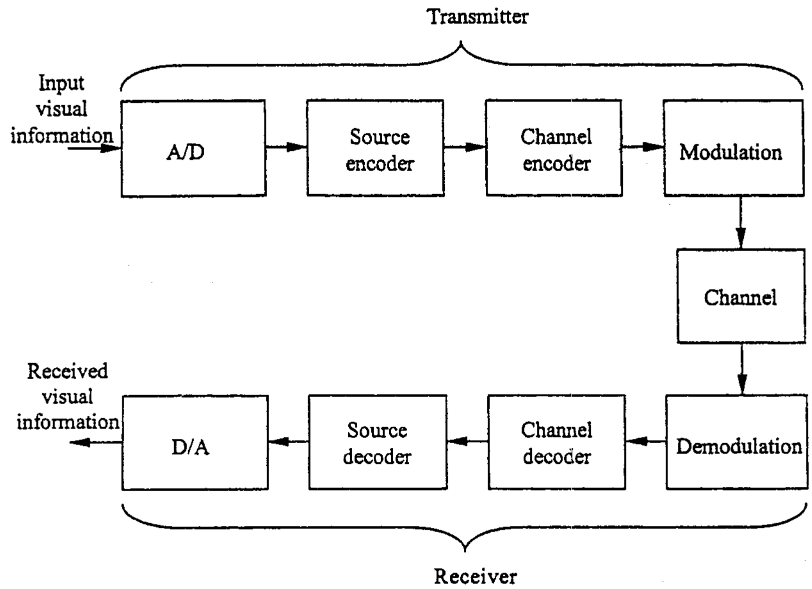

FIGURE 2.1 Block diagram of a visual communication system.

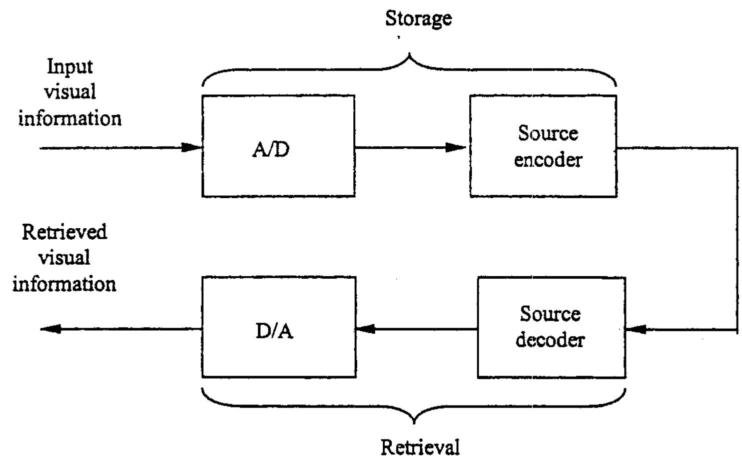

FIGURE 2.2 Block diagram of a visual storage system.

Hence, quantization is essentially discretization in magnitude, which is an important step in the lossy compression of digital image and video. (The reason that the term lossy compression is used here will be shown shortly.) The input and output of quantization can be either scalars or vectors. The quantization with scalar input and output is called *scalar quantization*, whereas that with vector input and output is referred to as *vector quantization*. In this chapter we discuss scalar quantization. Vector quantization will be addressed in Chapter 9.

After quantization, codewords are assigned to the many finitely different values from the output of the quantizer. Natural binary code (NBC) and variable-length code (VLC), introduced in Chapter 1, are two examples of this. Other examples are the widely utilized entropy code (including Huffman code and arithmetic code), dictionary code, and run-length code (RLC) (frequently used in facsimile transmission), which are covered in Chapters 5 and 6.

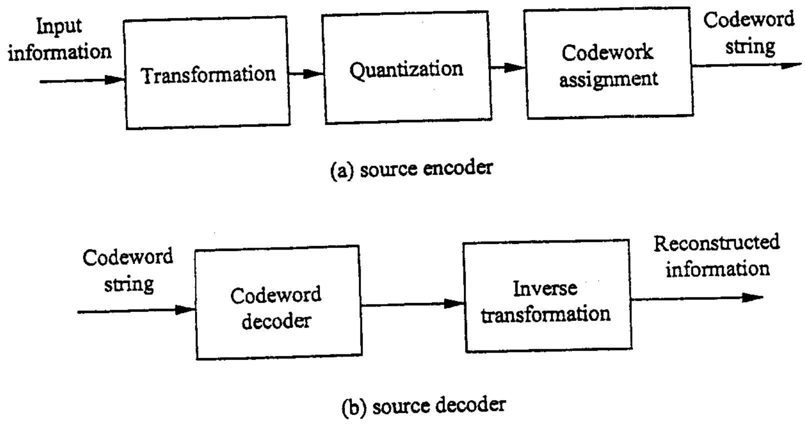

FIGURE 2.3 Block diagram of a source encoder and a source decoder.

The source decoder, as shown in Figure 2.3(b), consists of two blocks: codeword decoder and inverse transformation. They are counterparts of the codeword assignment and transformation in the source encoder. Note that there is no block that corresponds to quantization in the source decoder. The implication of this observation is the following. First, quantization is an irreversible process. That is, in general there is no way to find the original value from the quantized value. Second, quantization, therefore, is a source of information loss. In fact, quantization is a critical stage in image and video compression. It has significant impact on the distortion of reconstructed image and video as well as the bit rate of the encoder. Obviously, coarse quantization results in more distortion and a lower bit rate than fine quantization.

In this chapter, uniform quantization, which is the simplest yet the most important case, is discussed first. Nonuniform quantization is covered after that, followed by optimum quantization for both uniform and nonuniform cases. Then a discussion of adaptive quantization is provided. Finally, pulse code modulation (PCM), the best established and most frequently implemented digital coding method involving quantization, is described.

2.2 UNIFORM QUANTIZATION

Uniform quantization is the simplest and most popular quantization technique. Conceptually, it is of great importance. Hence, we start our discussion on quantization with uniform quantization. Several fundamental concepts of quantization are introduced in this section.

2.2.1 Basics

This subsection concerns several basic aspects of uniform quantization. These are some fundamental terms, quantization distortion, and quantizer design.

2.2.1.1 Definitions

Take a look at Figure 2.4. The horizontal axis denotes the input to a quantizer, while the vertical axis represents the output of the quantizer. The relationship between the input and the output best characterizes this quantizer; this type of configuration is referred to as the input-output characteristic of the quantizer. It can be seen that there are nine intervals along the x-axis. Whenever the input falls in one of the intervals, the output assumes a corresponding value. The input-output characteristic of the quantizer is staircase-like and, hence, clearly nonlinear.

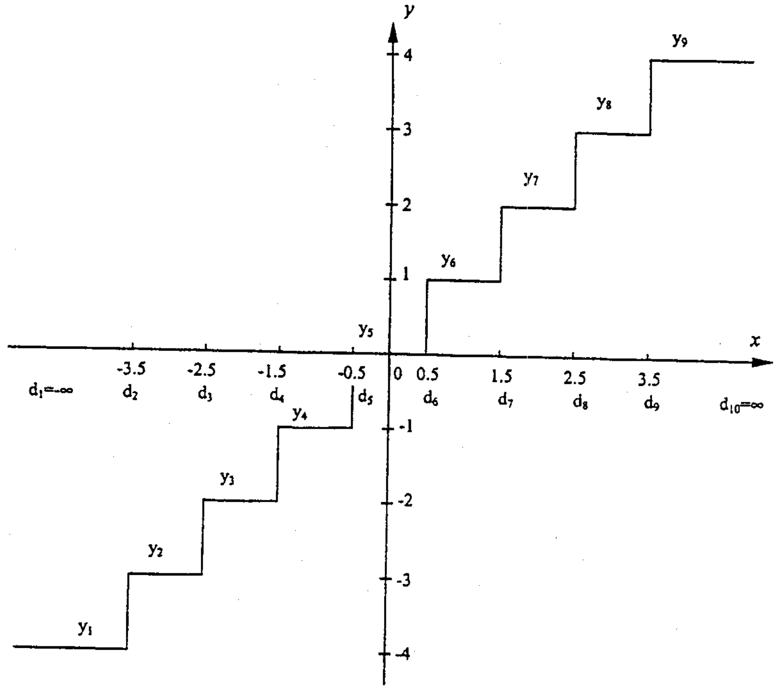

FIGURE 2.4 Input-output characteristic of a uniform midtread quantizer.

The end points of the intervals are called *decision levels*, denoted by d_i with i being the index of intervals. The output of the quantization is referred to as the *reconstruction level* (also known as *quantizing level* [Musmann, 1979]), denoted by y_i with i being its index. The length of the interval is called the *step size* of the quantizer, denoted by Δ. With the above terms defined, we can now mathematically define the function of the quantizer in Figure 2.4 as follows.

$$y_i = Q(x) \quad \text{if} \quad x \in \left(d_i,\, d_{i+1}\right) \tag{2.1}$$

where $i = 1,2,\cdots,9$ and $Q(x)$ is the output of the quantizer with respect to the input x.

It is noted that in Figure 2.4, $\Delta = 1$. The decision levels and reconstruction levels are evenly spaced. It is a uniform quantizer because it possesses the following two features.

1. Except for possibly the right-most and left-most intervals, all intervals (hence, decision levels) along the x-axis are uniformly spaced. That is, each inner interval has the same length.
2. Except for possibly the outer intervals, the reconstruction levels of the quantizer are also uniformly spaced. Furthermore, each inner reconstruction level is the arithmetic average of the two decision levels of the corresponding interval along the x-axis.

The uniform quantizer depicted in Figure 2.4 is called *midtread* quantizer. Its counterpart is called a *midrise* quantizer, in which the reconstructed levels do not include the value of zero. A midrise quantizer having step size $\Delta = 1$ is shown in Figure 2.5. Midtread quantizers are usually utilized for an odd number of reconstruction levels and midrise quantizers are used for an even number of reconstruction levels.

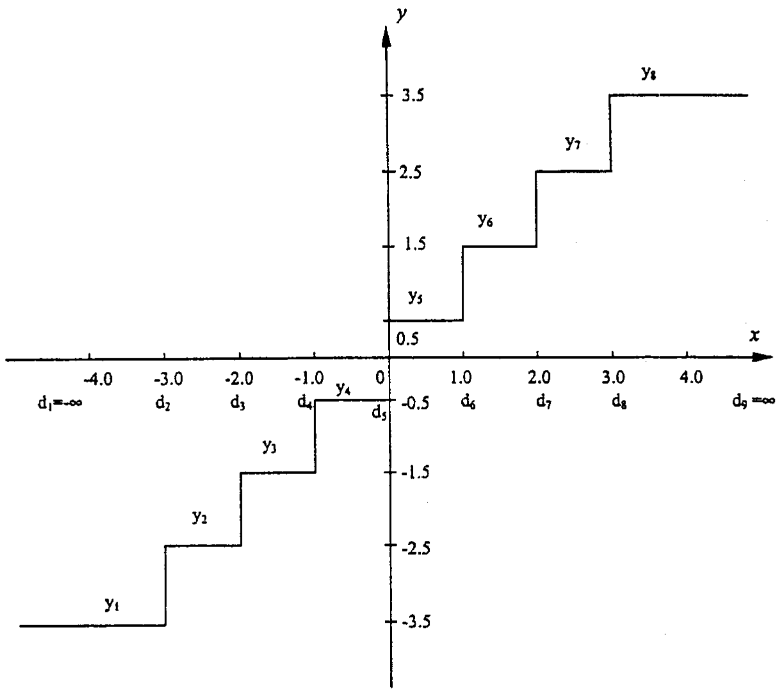

FIGURE 2.5 Input-output characteristic of a uniform midrise quantizer.

Note that the input-output characteristic of both the midtread and midrise uniform quantizers as depicted in Figures 2.4 and 2.5, respectively, is odd symmetric with respect to the vertical axis $x = 0$. In the rest of this chapter, our discussion develops under this symmetry assumption. The results thus derived will not lose generality since we can always subtract the statistical mean of input x from the input data and thus achieve this symmetry. After quantization, we can add the mean value back.

Denote by N the total number of reconstruction levels of a quantizer. A close look at Figure 2.4 and 2.5 reveals that if N is even, then the decision level $d_{(N/2)+1}$ is located in the middle of the input x-axis. If N is odd, on the other hand, then the reconstruction level $y_{(N+1)/2} = 0$. This convention is important in understanding the design tables of quantizers in the literature.

2.2.1.2 Quantization Distortion

The source coding theorem presented in Chapter 1 states that for a certain distortion D, there exists a rate distortion function $R(D)$, such that as long as the bit rate used is larger than $R(D)$ then it is possible to transmit the source with a distortion smaller than D. Since we cannot afford an infinite bit rate to represent an original source, some distortion in quantization is inevitable. In other words, we can say that since quantization causes information loss irreversibly, we encounter *quantization error* and, consequently, an issue: how do we evaluate the quality or, equivalently, the distortion of quantization. According to our discussion on visual quality assessment in Chapter 1, we know that there are two ways to do so: subjective evaluation and objective evaluation.

In terms of subjective evaluation, in Section 1.3.1 we introduced a five-scale rating adopted in CCIR Recommendation 500-3. We also described the false contouring phenomenon, which is caused by coarse quantization. That is, our human eyes are more sensitive to the relatively uniform regions in an image plane. Therefore an insufficient number of reconstruction levels results in